Comments on "Observation of Strong Quantum Depletion in a Gaseous Bose-Einstein Condensate" [cond-mat/0601184]

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A recent paper by K. Xu *et al.* [1] discussed a quantum depletion in gaseous Bose-Einstein condensation. In the ground state of an interacting Bose gas, not all the particles are in the lowest energy state, because the two-body interaction mixes the ground state components with atoms in the other states. For a homogeneous system the particle depletion is known well from Bogoliubov theory as [2] $\eta_0 = \frac{8}{3\sqrt{\pi}}\sqrt{\rho a^3}$, where ρ is the atomic density and a is the s-wave scattering length. The authors put the data of ^{23}Na into above relation and say that "For ^{23}Na at a typical density of $10^{14}cm^{-3}$, the quantum depletion is 0.2%". Apparently, this is not a reliable value for the inhomogeneous system which a magneto-optical trap is applied. For an interacting and inhomogeneous system the difficulty comes from the fact that we do not know the every energy eigenvalue of the system except ε_0 which is obtained by a macroscopic mean field wave function so called Gross-Piataevskii equation(GPE). However, one of the authors, W. Ketterle, already discussed the problem in an inhomogeneous system seriously and obtained an effective method for that [3]. We'll apply this method to obatin a better prediction.

We first consider a 3D system of N non-interacting Bosons confined by a harmonic potential of angular frequency ω. When the Bose distribution function in the grand canonical ensemble at temperature T is $f_B(\varepsilon)$ and the density of states of the isotropic 3D harmonic oscillator is $\rho(\varepsilon)$, the number of particles at temperature T is obtained by $N = \int f_B(\varepsilon)\rho(\varepsilon)d\varepsilon$ [3, 4, 5, 6]. It is

$$N = N_0 + \left(\frac{k_B T}{\hbar\omega}\right)^3 g_3(z) + \frac{2}{3}\left(\frac{k_B T}{\hbar\omega}\right)^2 g_2(z) + ..., \quad (1)$$

where $g_n(z) = \sum_{l=1}^{\infty} z^l/l^n$ is the Bose functions. $z = e^{\beta(\mu-\varepsilon_0)}$ is the fugacity between 0 and 1, $\varepsilon_0 = (3/2)\hbar\omega$ is the zero-point energy of the harmonic oscillator, and $\mu(T)$ is the chemical potential. N_0 is the ground state occupation number given by $N_0 = 1/(z^{-1} - 1)$. The others are common notations.

For interacting system there is a shift of the zero point energy by the interaction. Then, the new ground state occupation number is written as

$$N_0^{int} = \frac{1}{e^{\beta(\varepsilon_0 + U - \mu)} - 1}, \quad (2)$$

where $U = U(a, N_0)$ is the interaction energy between atoms and may obtained from the solution of the GPE. We consider the positive scattering length only, so does $U > 0$. Then, the condensate fraction of the interacting system is definitely lower than that of the ideal system.

The chemical potential $\mu(T)$ is obtained numerically by the following way. The transition temperature T_c is determined that at the onset of condensation $N_0 \to 0$ and $z \to 1$ from Eq. (1). It is

$$T_c = T_0\left\{1 - \frac{\zeta(2)}{2\zeta(3)^{2/3}}\frac{1}{N^{1/3}} + ...\right\}, \quad (3)$$

where $T_0 = (\hbar\omega/k_B)\{N/\zeta(3)\}^{1/3}$ and ζ is the Riemann-zeta function. $T_c = 328 \sim 713 nK$ for $N = 10^5 \sim 10^6$ and $\omega = 10^3 sec^{-1}$. The condensate fraction is obtained from Eq. (1) as a function of temperature [3, 4, 5, 6]

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_0}\right)^3 - \frac{3\zeta(2)}{2\zeta(3)^{2/3}N^{1/3}}\left(\frac{T}{T_0}\right)^2 + ... \quad (4)$$

The author's prediction of the quantum depletion 0.2% provides that the transition temperature should be below $40 \sim 90 nK$ for $N = 10^5 \sim 10^6$. Actually, the author's estimated value $\eta_0 = 0.2\%$ is too small for the inhomogeneous system to be accepted. We obtain the fugacity $z(T)$ first by solving Eq. (4) numerically, and then obtain the chemical potential $\mu(T)$. We use this $\mu(T)$ for the estimation of N_0^{int}.

Comparing N_0 and N_0^{int}, the over all behavior of the condensate fraction in Eq. (2) as a function of scattering length and temperature can be estimated by the approximation

$$N_0^{int}(a, T) \sim N_0(T)e^{-\beta U(a, N_0)}. \quad (5)$$

The ground state occupation number falls exponentially as the interaction increases.

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